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Direct and Inverse Dynamic Problems for a System of Equations of Homogeneous Elastic-Porous Media

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Abstract—A solution to a system of equations of elastic-porous media for a homogeneous space in the time and frequency domains for the case of a point source is obtained. Ray representations of waves of various types in an inhomogeneous elastic-porous medium are obtained. Inverse problems of determining the parameters of the medium and seismic moment tensor are considered. This is done by using the information about

(1) component parts of the displacement vectors of P_1- and S-waves at a fixed point of space;
(2) pressure measured at two fixed points of space for all times.

The noise stability of the solutions to the inverse problems considered is investigated numerically with the use of the method of critical components.

Keywords—Mathematical model, Porous medium, Method of critical component, Inverse problem.

1. INTRODUCTION

In applied problems of elastic wave propagation, it is often necessary to take into account porosity, the fluid saturation of the medium, and the hydrodynamic background. In particular, these questions arise in exploration geophysics, in the search for oil formations and when choosing the

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parameters of wave action on oil and gas deposits to intensify production. Similar questions arise in seismology at geophysical monitoring of the properties of the source zone for the purpose of earthquake prediction.

In geophysics, the dynamic and kinematic characteristics of elastic waves propagating in fragmented fluid-saturated rocks contain information about the structure, composition, and occurrence of rocks. Also, they have information about the lithology of rocks and their boundaries, fracturing, porosity, the presence of various faults and local inclusions, the composition and phase state of fluids-fillers of the collectors' pore space. Mathematical models of wave propagation theory give an instrument to determine the numerical values of the propagation speeds and absorption coefficients of elastic seismic waves depending on the substances of the fluid-filled collector, its structure, and the influence of the environment. The more realistic and reliable the mathematical model, the more accurate the determined values of propagation speed and the absorption coefficient of elastic seismic waves.

The peculiarities of seismic wave absorption in fractured-porous media that have been established by now and the simultaneous manifestation of multiple electroseismic effects are not consistent with the simplest models of the ideally elastic Hooke and Frenkel-Biot media. Real geological media are multiphase, conducting, fractured, porous, etc. In the process of propagation, seismic waves dissipate, which is associated with energy absorption.

In this paper, modeling is performed in terms of c_s, c_{p_1}, and c_{p_2} and in terms of the partial densities ρ_l and ρ_s, with the use of the fact of established existence of one-to-one correspondence between the elastic parameters λ, μ, and α of the theory developed by Dorovsky [1,2] and the propagation speeds of seismic waves c_s, c_{p_1}, and c_{p_2} in elastic-porous media.

In the second section, we obtain a solution to the linearized system of Dorovsky's equations for the entire homogeneous space in the frequency domain in the case when wave processes are caused by a point source of the concentrated force type.

In the third section, ray expansion for a concentrated force field in infinite homogeneous and inhomogeneous media is obtained.

In the fourth section, to determine the seismic moment tensor, we use, in contrast to [3–9], a part of the components of the displacement vectors of the P_1- and S-waves measured only at one point of space for all frequencies.

In the fifth section, to determine the seismic moment tensor, we use the pressures measured only at two points of space for all frequencies.

The noise stability of the solutions to the inverse problems considered is investigated numerically with the use of the method of critical components [10], realized in the form of a program package called JINRLINPACK [11].

2. DIRECT DYNAMIC PROBLEM FOR EQUATIONS OF CONTINUAL FILTRATION THEORY AND THEORY OF ELASTIC WAVES

A nonlinear mathematical model of filtration of a liquid in a porous elastic-deformed medium is constructed in [1]. The following three principles are used as a basis of the model: fulfillment of conservation laws, the Galilei relativity principle, the agreement between the equations of motion and the thermodynamic conditions of equilibrium. It is shown that the system of equations linearized with respect to an arbitrary hydrodynamic background in a reversible hydrodynamic approximation (in the absence of energy dissipation) is hyperbolic. This linearized system of equations for the homogeneous medium has the following form [12,13]:

$$\frac{\partial^2 \mathbf{U}}{\partial t^2} - c_s^2 \Delta \mathbf{U} - \left(a_1 - c_s^2\right) \nabla \operatorname{div} \mathbf{U} + a_2 \nabla \operatorname{div} \mathbf{V} = \mathbf{f}, \tag{1}$$

$$\frac{\partial^2 \mathbf{V}}{\partial t^2} - a_4 \nabla \operatorname{div} \mathbf{V} + a_3 \nabla \operatorname{div} \mathbf{U} = \mathbf{f}, \tag{2}$$

Here, $\mathbf{U}$ and $\mathbf{V}$ are the displacement vector of particles of the elastic porous body and liquid with the partial densities ρ_s and ρ_l, respectively, and $\mathbf{f}$ is the body force.

The main difference of linearized Dorovsky's model from the well-known Frenkel-Biot models [14,15] is that Dorovsky's model is described by three elastic constants [1]. It is shown in [2] that the coefficients a_k, $k = \overline{1,4}$, are expressed in terms of the speed c_s of the transverse wave, the speed c_{p_m} $(m = 1, 2)$ of longitudinal waves, as well as the ratios between the partial density ρ_l of the conducting liquid and the partial density ρ_s of the conducting elastic porous body as follows:

$$a_1 = \frac{\rho_l}{\rho}\left(c_{p_1}^2 + c_{p_2}^2\right) + \frac{4}{3}\frac{\rho_s^2}{\rho^2}c_s^2 + \frac{\rho_s - \rho_l}{\rho}\tilde{z}, \qquad a_2 = \frac{\rho_l}{\rho}\left(c_{p_1}^2 + c_{p_2}^2 - 2\tilde{z} - \frac{4}{3}c_s^2\right),$$

$$a_3 = \frac{\rho_s}{\rho}\left(c_{p_1}^2 + c_{p_2}^2 - 2\tilde{z} - \frac{4}{3}c_s^2\right), \qquad a_4 = \frac{\rho_s}{\rho}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{4}{3}c_s^2\right) - \frac{\rho_s - \rho_l}{\rho}\tilde{z},$$

$$\tilde{z} = \frac{1}{2}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_s}{\rho}c_s^2\right) + \sqrt{\frac{1}{4}\left(c_{p_1}^2 - c_{p_2}^2\right)^2 - \frac{16}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}, \qquad \rho = \rho_s + \rho_l.$$

It should be noted that, in principle, the Frenkel-Biot theory cannot be used for statements of inverse problems and closed numerical modeling of various processes in the porous medium, when the distribution of seismic wave propagation speeds c_s, c_{p1}, and c_{p2} and the densities ρ_l and ρ_s are known [2].

In the construction of the solution to the direct problems, we consider a source of point type (spherical wave), that is,

$$f_\nu(t, \mathbf{x}) = \mathrm{M}_{\nu\beta}(t)\frac{\partial\delta(\mathbf{x} - \mathbf{y})}{\partial x_\beta}. \tag{3}$$

Here, $\mathrm{M}_{\alpha\beta}(t) = \mathrm{M}_{\beta\alpha}(t)$ is the seismic moment tensor [16], $\delta(t)$ is the Dirac function, $\mathbf{x} = (x_1, x_2, x_3)$ is the coordinate of the receiver, and $\mathbf{y} = (y_1, y_2, y_3)$ is the coordinate of the source.

The solution to equations (1),(2) with the right-hand side (3) in the frequency domain has the following form:

$$\begin{aligned}\hat{\mathbf{U}}_\alpha(\omega, \mathbf{x}) = \hat{\mathbf{U}}_\alpha^s(\omega, \mathbf{x}) &- \frac{a_4 - c_{p_1}^2}{a_3}\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x}) \\ &+ \frac{a_4 - c_{p_2}^2}{a_3}\frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}),\end{aligned} \tag{4}$$

$$\hat{\mathbf{V}}_\alpha(\omega, \mathbf{x}) = -\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x}) + \frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}), \tag{5}$$

$$\hat{\mathbf{U}}_\alpha^s(\omega, \mathbf{x}) = \Delta\frac{\partial\hat{\mu}_{\alpha\beta}(\omega, c_s, \mathbf{x})}{\partial x_\beta} - \frac{\partial^3\hat{\mu}_{\nu\beta}(\omega, c_s, \mathbf{x})}{\partial x_\alpha\partial x_\beta\partial x_\nu},$$

$$\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x}) = \frac{\partial^3\hat{\mu}_{\nu\beta}(\omega, c_{p_1}, \mathbf{x})}{\partial x_\alpha\partial x_\beta\partial x_\nu}, \qquad \hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}) = \frac{\partial^3\hat{\mu}_{\nu\beta}(\omega, c_{p_2}, \mathbf{x})}{\partial x_\alpha\partial x_\beta\partial x_\nu},$$

$$\hat{\mu}_{\alpha\beta}(\omega, c, \mathbf{x}) = \hat{\mathrm{M}}_{\alpha\beta}(\omega)\frac{\mathrm{e}^{-i(\omega/c)|\mathbf{x}-\mathbf{y}|}}{4\pi\omega^2|\mathbf{x}-\mathbf{y}|}, \qquad |\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

The solution to the problem for more general types of sources is constructed with the help of the superposition of the fields from the point source.

It should be noted that when porosity vanishes, i.e., as $\rho_l \to 0$, solution (4) is transformed into the solution to the Lamé system of equations for a homogeneous medium [17,18].

3. RAY EXPANSION FOR THE CONCENTRATED FORCE FIELD IN AN INFINITE MEDIUM

It was shown in [19] that when the source field in a homogeneous isotropic elastic medium is known, one can determine the intensity and form of representation in the zero approximation for waves propagating in an inhomogeneous medium, whose elastic parameters at the source point are the same as those of the homogeneous medium.

In this section, we present formulas illustrating the character of attenuation and polarization of the displacement vectors of particles of the elastic porous body and the liquid in subsequent approximations using the approach proposed in [19].

Let a source of force type be applied at the origin of coordinates. The source acts along the Oz-axis and varies with time as $f_0(t)$.

Let us represent the displacement values of particles of the elastic porous body in the form [20]:

$$\mathbf{U} = \mathbf{U}_s - \frac{a_4 - c_{p_1}^2}{a_3}\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\mathbf{U}_{p_1} + \frac{a_4 - c_{p_2}^2}{a_3}\frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\mathbf{U}_{p_2},$$

$$\mathbf{V} = -\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\mathbf{U}_{p_1} + \frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\mathbf{U}_{p_2}.$$

Here, the first term $\mathbf{U}_s$ determines the field of the excited transverse wave. The displacement vector of particles of this wave is perpendicular to the direction $\mathbf{R}_1$ of its propagation ($\mathbf{R}_1$ and θ_1 are the basis vectors in the spherical system of coordinates). The fronts coincide with the spheres $c_s t - R = \text{const}$, whose radii increase at the speed c_s. The second and third terms correspond to longitudinal waves. The displacements of particles of these waves are parallel to the direction $\mathbf{R}_1$ of their propagation. The fronts coincide with the spheres $c_{p_k}t - R = \text{const}$, $(k = 1, 2)$, whose radii increase at the speed c_{p_k}.

Repeating the reasoning of [19], we obtain the following expressions for the vectors $\mathbf{U}_s$, $\mathbf{U}_{p_1}$, and $\mathbf{U}_{p_2}$:

$$\mathbf{U}_s = \mathbf{U}_s^{(0)} f_0(t-\tau_s) + \mathbf{U}_s^{(1)} f_1(t-\tau_s) + \mathbf{U}_s^{(2)} f_2(t-\tau_s), \qquad \tau_s = \frac{R}{c_s},$$

$$\mathbf{U}_{p_k} = \mathbf{U}_{p_k}^{(0)} f_0(t-\tau_{p_k}) + \mathbf{U}_{p_k}^{(1)} f_1(t-\tau_{p_k}) + \mathbf{U}_{p_k}^{(2)} f_2(t-\tau_{p_k}), \quad \tau_{p_k} = \frac{R}{c_{p_k}}, \qquad k = 1, 2,$$

$$\mathbf{U}_{p_k}^{(0)} = \frac{\cos\theta}{4\pi\rho_s c_{p_k}^2 R}\mathbf{R}_1, \qquad \mathbf{U}_{p_k}^{(1)} = \frac{2\cos\theta}{4\pi\rho_s c_{p_k} R^2}\mathbf{R}_1 + \frac{\sin\theta}{4\pi c_{p_k} R^2}\theta_1,$$

$$\mathbf{U}_{p_k}^{(2)} = \frac{2\cos\theta}{4\pi\rho_s R^3}\mathbf{R}_1 + \frac{\sin\theta}{4\pi\rho_s R^3}\theta_1,$$

$$\mathbf{U}_s^{(0)} = -\frac{\sin\theta}{4\pi\rho_s c_s^2 R}\theta_1, \qquad \mathbf{U}_s^{(1)} = -\frac{2\cos\theta}{4\pi\rho_s c_s R^2}\mathbf{R}_1 - \frac{\sin\theta}{4\pi c_s R^2}\theta_1,$$

$$\mathbf{U}_s^{(2)} = -\frac{2\cos\theta}{4\pi\rho_s R^3}\mathbf{R}_1 - \frac{\sin\theta}{4\pi\rho_s R^3}\theta_1,$$

$$f_1(t) = \int_0^t f_0(s)\,ds, \qquad f_2(t) = \int_0^t f_1(s)\,ds.$$

Using the same line of reasoning as in [19,20], we obtain a ray solution to Dorovsky's system of equations for the inhomogeneous medium:[1]

$$\mathbf{U}(t,x) = \sum_{k=0}^{\infty} U_k(t,x) f_k(\gamma(t,x)),$$

$$\mathbf{V}(t,x) = \sum_{k=0}^{\infty} V_k(t,x) f_k(\gamma(t,x)),$$

$$f_k(x) = \int f_{k-1}(x)\,dx, \qquad k = 1, 2, 3, \dots,$$

[1]The series are of asymptotic character. At the assumption of analyticity of the coefficients and functions $\gamma(t,x)$, they converge uniformly in the vicinity of the surfaces $\gamma(t,x) = 0$.

where $\gamma(t,x)$ is a fixed function, and for any k, $\mathbf{U}_k$, and $\mathbf{V}_k$ the recurrent relations

$$\mathbf{N}(\gamma, \mathbf{U}_{k+2}, \mathbf{V}_{k+2}) + \mathbf{M}(\gamma, \mathbf{U}_{k+1}, \mathbf{V}_{k+1}) + \tilde{\mathbf{L}}(\gamma, \mathbf{U}_k, \mathbf{V}_k) = 0,$$
$$(\mathbf{U}_{-2} = \mathbf{V}_{-2} = \mathbf{U}_{-1} = \mathbf{V}_{-1} \equiv 0), \qquad k = -2, -1, 0, 1, 2, \ldots,$$

are satisfied. Here we omit the operators $\tilde{\mathbf{L}}$, $\mathbf{N}$, and $\mathbf{M}$ presented in [21], because they are too cumbersome.

It is also not difficult (but cumbersome) to construct a theory of plane waves for a multilayered homogeneous medium with sharp interfaces between layers. The reflection-refraction coefficients of waves of all three types are necessary for the realization of the ray method in the inhomogeneous medium. Local representations of the fields of point sources for the construction of ray series are also required.

4. INVERSE PROBLEM OF DETERMINING THE SEISMIC MOMENT TENSOR BY USING THE INFORMATION ABOUT ARRIVAL TIMES OF LONGITUDINAL AND TRANSVERSE WAVES

Let the arrival times of longitudinal and transverse waves at some points of space and the coordinates of a seismic event be known. With the use of this information, it is necessary to determine the seismic moment tensor. Mathematically, the problem is formulated as follows: it is necessary to find $\mathrm{M}_{\alpha\beta}(t)$ with the help of the following information:

$$U^{p_1}_{\alpha'}(t,\mathbf{x})\big|_{\mathbf{x}=\mathbf{x}_0} = U^{p_1}_{\alpha'}(t), \tag{6}$$

$$U^{s}_{\beta'}(t,\mathbf{x})\big|_{\mathbf{x}=\mathbf{x}_0} = U^{s}_{\beta'}(t). \tag{7}$$

Here, α' and β' are fixed numbers, which take different values, from 1 to 3.

Let the seismic moment tensor $\mathrm{M}_{\alpha\beta}(t)$ have the following form [7]:

$$\mathrm{M}_{\alpha\beta}(t) = a \cdot \delta(t) \cdot \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}, \tag{8}$$

where $a = 10^{19}$ dyne·cm·s.

Since the solution to system of equations (1),(2) is given in the frequency domain, the solution to the inverse problem will also be realized in the frequency domain. For this, we perform the Fourier transform of the data (6) and (7). Assuming in (4) that $\mathbf{x} = \mathbf{x}_0$ and using the notation $\mathbf{Z} = (\hat{\mathrm{M}}_{11}(\omega), \hat{\mathrm{M}}_{22}(\omega), \hat{\mathrm{M}}_{33}(\omega), \hat{\mathrm{M}}_{12}(\omega), \hat{\mathrm{M}}_{13}(\omega), \hat{\mathrm{M}}_{23}(\omega))$, we obtain

$$\mathrm{A}\mathbf{Z} = \mathbf{Y}. \tag{9}$$

Here, $\mathbf{Y} = \mathrm{Re}(\hat{U}^{p_1}_{\alpha'}(\omega,\mathbf{x}_0), \hat{U}^{s}_{\beta'}(\omega,\mathbf{x}_0))^{\top}$ and, by virtue of (8), the components of the matrix $\mathrm{A} = (A_{ij})_{2\times 6}$ are real.

Thus, the solution to the inverse problem is reduced to the solution of system of linear algebraic equations (9).

Let the pore pressures at some points $\mathbf{x}_k$, $k = 1, 2$, of space be known. By these measurements, it is required to determine the seismic moment tensor. Mathematically, the problem in terms of the Fourier transform is formulated as follows: it is necessary to find $\hat{\mathrm{M}}_{\alpha\beta}(\omega)$ by using the information

$$\hat{P}(\omega,\mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_k} = \hat{P}_k(\omega), \qquad k = 1, 2,$$

where $P = \rho\,\mathrm{div}(a_3\mathbf{U} - a_4\mathbf{V})$, and the coordinates of the source are known.

Assuming in (4),(5) that $\mathbf{x} = \mathbf{x}_0$ and taking into account the pore pressure definition for $\mathbf{Z}$, we obtain

$$\tilde{\mathrm{A}}\mathbf{Z} = \tilde{\mathbf{Y}}, \tag{10}$$

where $\tilde{\mathbf{Y}} = \mathrm{Re}(\hat{P}_1(\omega), \hat{P}_2(\omega))^{\top}$ and the components of the matrix $\tilde{\mathrm{A}} = (\tilde{A}_{ij})_{2\times 6}$ are real.

5. NUMERICAL SOLUTIONS OF SYSTEMS (9) AND (10) BY USING THE METHOD OF CRITICAL COMPONENTS

To solve ill-posed systems of linear algebraic equations ((9) and (10)), we use the method of critical components [10]. $\tilde{\mathbf{Z}}$, approximate solutions to system (9) and (10), respectively, are given below in Tables 1 and 2. The relative solution errors $\delta = \|\mathbf{Z} - \tilde{\mathbf{Z}}\|/\|\mathbf{Z}\|$ at the corresponding relative perturbations of the right-hand sides $\varepsilon = \|\mathbf{Y} - \tilde{\mathbf{Y}}\|/\|\mathbf{Y}\|$ are also given. The parameters of the medium take the following values: $c_{p_1} = 6.1$ km/s, $c_{p_2} = 0.6$ km/s, $c_s = 3.5$ km/s, $\rho_s = \rho_s^f(1 - d_0)$, $\rho_l = \rho_l^f d_0$, $\rho_s^f = 2.7$ g/cm^3, $\rho_l^f = 0.9$ g/cm^3, $d_0 = 10\%$, $\omega = 10$ Hz, $\alpha' = \beta' = 1$.

Table 1.

ε	$\tilde{Z}_1$	$\tilde{Z}_2$	$\tilde{Z}_3$	$\tilde{Z}_4$	$\tilde{Z}_5$	$\tilde{Z}_6$	δ
0%	1.400	2.900	2.900	4.300	4.300	5.800	13.3%
10%	1.540	3.190	3.190	4.730	4.730	6.380	16.6%
20%	1.680	3.480	3.480	5.160	5.160	6.960	23.8%
30%	1.820	3.770	3.770	5.590	5.590	7.540	32.6%
40%	1.960	4.060	4.060	6.020	6.020	8.120	41.8%

Table 2.

ε	$\tilde{Z}_1$	$\tilde{Z}_2$	$\tilde{Z}_3$	$\tilde{Z}_4$	$\tilde{Z}_5$	$\tilde{Z}_6$	δ
0%	0.930	2.496	2.496	4.796	4.976	4.992	15.5%
10%	1.023	2.746	2.746	5.276	5.276	5.491	16.9%
20%	1.116	2.995	2.995	5.755	5.755	5.990	22.9%
30%	1.209	3.245	3.245	6.235	6.235	6.490	30.4%
40%	1.302	3.494	3.494	6.715	6.715	6.989	39.0%

It is seen from Tables 1 and 2 that the error of reconstruction of components of the seismic moment tensor is at the data error level.

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